

The relationship of creep, recovery and fatigue in polycarbonate

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The quantitative relationship between creep and recovery which had been previously developed by Mindel and Brown [1] has been applied to interrupted creep tests. Single and multiple interruptions (fatigue) were investigated. In general it was observed that interruptions decreased the time to failure. The experiments were conducted in compression in the range of high stresses. Failure was caused by excessive deformation or accelerated creep which is produced by a softening mechanism that is common to all linear polymers. The temperature changes associated with the creep and recovery parts of the cyclic loading were measured. The temperature rise during loading exceeds the decrease during unloading so that there is a net rise in temperature if the creep and recovery intervals are equal. However, the temperature change is not the primary cause for the decrease in time-to-failure for cyclic loading as compared to the failure time during steady stress creep. It has been concluded that fatigue failure under compressive deformation is related fundamentally to the constant stress creep curve.

1. Introduction

In a previous paper, Mindel and Brown [1] investigated the relationship between creep and recovery of polycarbonate in the region of high creep stresses where non-linear viscoelasticity prevails. M-B [1] also investigated the critical condition under which creep failure occurred and found that under compressive loading the PC began to fail by an acceleration of the creep rate after a total strain of about 8.8%. The question was then raised concerning the conditions for creep failure if the creep test was interrupted by a period of recovery. Since a sequence of creep, recovery and creep can be viewed as one cycle of fatigue loading, the present investigation was extended from one cycle, to several cycles, and then to many cycles. It is our goal to determine whether the behaviour after many cycles can be interpreted in terms of the single creep and recovery process.

There is much evidence that indicates that failure in PC occurs by a softening process that is associated with deformation. Rabinowitz and Beardmore [2] observed a "softening" in a wide range of polymers after cyclic loading at a fixed strain well below the yield strain; the softening was observed by a decrease in the yield point of

the polymer. Tonkins and Biggs [3] observed cyclic softening in nylon. Imai and Higuchi [4] investigated the fatigue of PC and presented a criterion for failure based on the accumulation of a critical amount of softening; the softening was associated with a rise in temperature of the sample, but the rise in temperature *per se* was not the cause of failure. M-B [1] showed that the softening process which caused creep failure was not produced by the rise in temperature but by an increase in the amount of permanent strain which persisted during recovery. Compression data for various polymers [5, 6] show homogeneous deformation up to the yield point where a drop in the true stress occurs; this drop in stress indicates that yielding is associated with an intrinsic softening of the material.

In this investigation creep failure was investigated after the creep process was interrupted by one or more periods of recovery. Compressive loading was used to avoid crazing. The temperature of the specimen was measured to determine the extent to which thermal effects might be involved. It was found that the time to produce creep failure could be greatly decreased if the creep test was interrupted by one or more periods

of recovery and that the thermal effect was not the primary cause.

2. Experimental

2.1. Material

The polycarbonate was commercial "Lexan", and the specimens were cut from the identical rod that was used in the previous investigation [1]. The diameter of the rod was 0.5 in. from which specimens 0.550 in. in length were machined and whose ends were metallographically polished. The ends were sprayed with a Teflon lubricant before testing.

2.2. Procedures

The same experimental procedures were used as before [1]. Testing was performed on an MTS Systems closed loop testing machine between flat parallel anvils. Specimens were loaded in 0.1 sec, and strain readings were taken at 1 sec intervals. Strain rate was calculated using three points so that the initial creep rate as reported in this investigation corresponds to the second second after loading. The temperature measurements were made by inbedding thermocouples in 0.016 in. holes which were drilled half way into the specimens. Most tests were done without measuring the temperature.

3. Results

3.1. Uninterrupted creep

Fig. 1 is reproduced from our previous paper [1] and gives a base line for evaluating the subsequent data. The data will often be displayed as in Fig. 1 where $\ln \dot{\epsilon}$ is plotted versus ϵ , the total strain. This plot will be referred to as a Sherby-Dorn plot [7] (S-D). One advantage of the S-D plot is that all the curves can be brought together by a vertical shift and thus be described by the equation

$$\dot{\epsilon} = e^{(\sigma v - Q)/kT} f(\epsilon) \quad (1)$$

where σ is stress, v is a constant which is equal to 5700 \AA^3 for PC, and Q is the activation energy, $f(\epsilon)$ is a function of the total strain, ϵ . It is to be noted that accelerating creep or creep failure begins at a strain of 8.8% and is independent of the stress.

3.2. Single interruption of creep

In a single-interruption test, the specimen is crept for a specific time at a certain stress; it is recovered under zero stress for a specified time, and then it is reloaded to the same stress. Fig. 2 shows a typical result where the specimen was

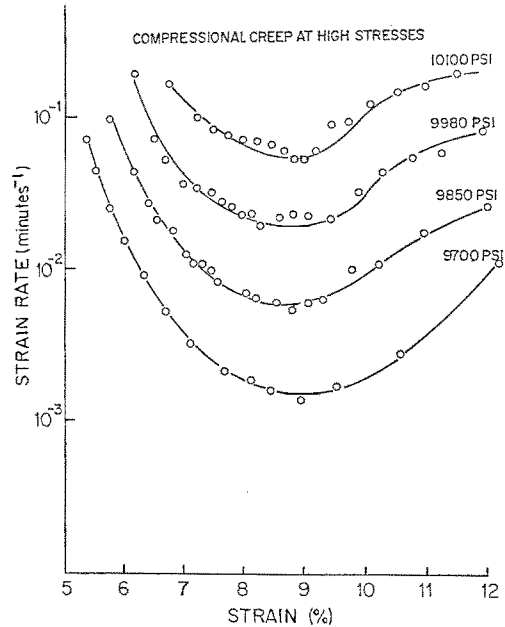


Figure 1 Log strain rate versus strain for various compressive stresses.

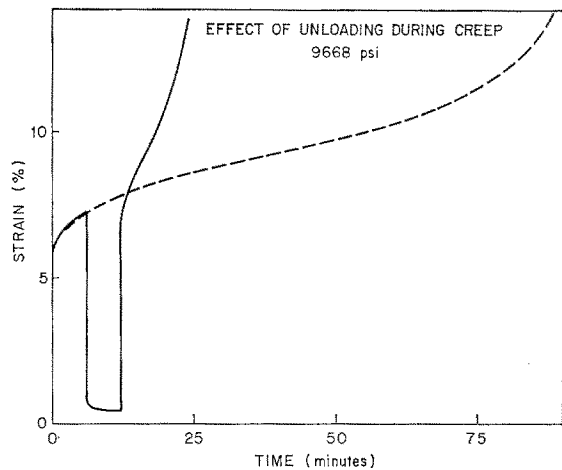


Figure 2 Effect of a single interruption of a creep test. Dotted line is the uninterrupted curve.

crept for 6 min and recovered for 6 min and then reloaded; if the specimen had not been unloaded, it would have begun to fail in 40 min, whereas it began to fail almost immediately after re-loading. It is to be noted that the total strain at which creep failure began was about the same, about 9%, for both the interrupted and uninterrupted tests.

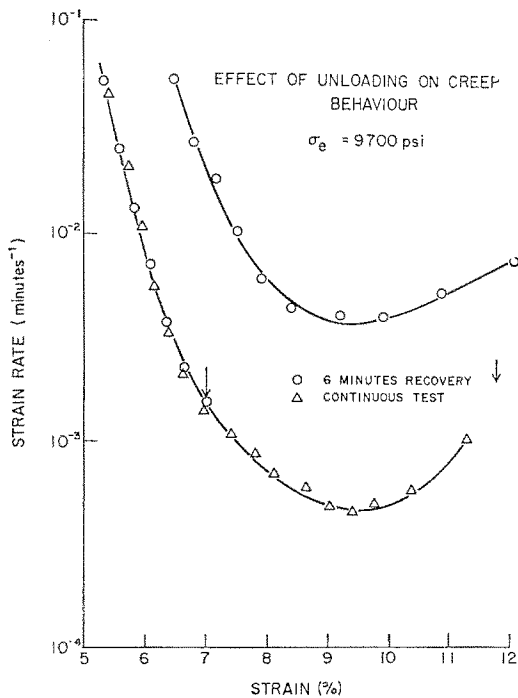


Figure 3 Log strain rate versus strain for an interrupted and uninterrupted creep test.

The results of this test are shown in Fig. 3 as an S-D plot. It is to be noted that, after reloading the specimen begins to creep at the same initial rate as for the first test. The shape of the S-D plot from the final creep test is similar to the initial one, and it is displaced upward. Since the specimen began the final creep test with a residual or permanent strain, and since the initial creep

rate for the second test is the same as the initially high creep rate for the first test, it is to be expected that the time for creep failure should be reduced. These results suggest that, when a total strain of about 9% is reached, creep failures begins whether or not recovery is permitted to take place.

3.3. Temperature effect

The temperature changes are shown in Fig. 4. The initial rise in temperature of about 1.2°C is partially from the thermoelastic effect and partially from the heat generated by the strain. Then the specimen loses heat to its surroundings faster than it is generated by the deformation process at the beginning of the creep test. It was shown by M-B [1] that heat generation during the softening processes was associated with the production of persistent strain. Most of the strain that is generated prior to the minimum in the S-D plot is recoverable. During unloading the specimen is cooled by the thermoelastic decompression. Note that the decrease in temperature during unloading is only 0.6° compared to a 1.2°C increase during loading. The times for loading and unloading were the same, 0.1 sec, and therefore, the difference between the temperature changes during loading and unloading cannot be attributed to a difference in the losses to the surroundings. In the previous investigation [1] it was found that the instantaneous strain during unloading was equal or greater than the instantaneous strain during unloading (see Figs. 2 and 4); apparently this asymmetry between the loading and unloading

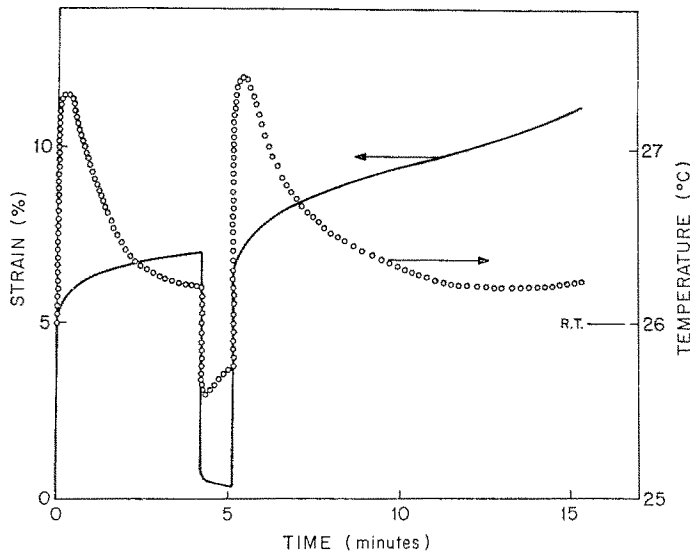


Figure 4 Creep curve with a single interruption which includes the simultaneous temperature measurement.

strain manifests itself as a thermal asymmetry. Possibly the difference between the instantaneous loading and unloading strains consists of an increment of strain which generates heat and thus reduces the thermoelastic temperature drop during unloading.

When the specimen was reloaded (Fig. 4) the peak temperature was about 1.3°C above ambient compared to 1.2°C for the initial loading. The additional 0.1°C rise in temperature is not sufficient to produce the increase in creep rate that was observed after reloading (Fig. 4). Note also that, as pointed out previously [1], the beginning of accelerated creep is not accounted for by the temperature changes that occur prior to the accelerating process.

3.4. Effect of recovery

The effect of the length of the recovery time on the subsequent creep behaviour was investigated. In the case of Fig. 5, all specimens were crept to a strain of about 7.5% under a stress of 9850 psi*. Each specimen was then unloaded to zero stress, recovered for a certain time, and then creep tested at 9850 psi. Recovery times of 1, 10, and 100 sec were used. The results are shown as S-D plots. All three specimens had the same

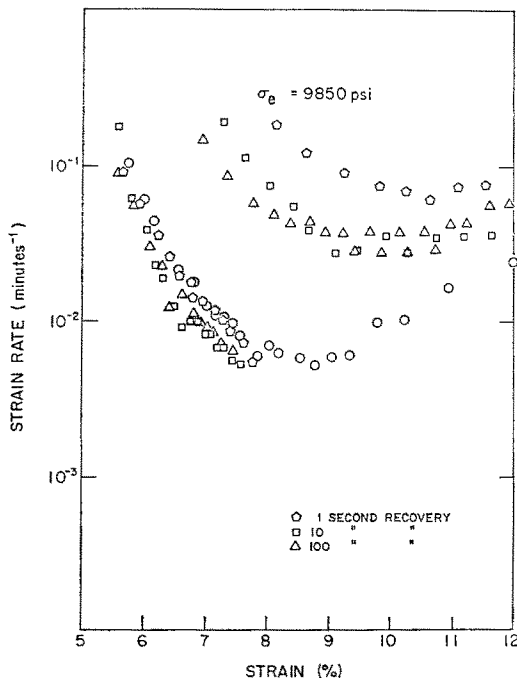


Figure 5 Log strain rate versus strain showing effect of recovery time.

* 10^3 psi = 6.89 N mm $^{-2}$.

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initial creep curve; also, the initial creep rates for their second creep test were all equal and were about the same as the initial creep rate for their first creep test. However, each specimen began to creep during the second test with a total strain that was about equal or less than the strain just prior to unloading. The difference between the strain prior to unloading and the initial strain during the second creep test increases with the recovery time. The S-D plot during the second creep test has about the same shape as the S-D plot from an uninterrupted test. In agreement with Fig. 3, there are three important features to be noted about the second creep test: (1) the second creep test starts at the same creep rate as that for the initial creep test (2) the creep rate decreases in accordance with S-D function, $f(\epsilon)$, with its starting point at $f(\epsilon_1)$ where ϵ_1 is the total strain immediately after reloading and (3) ϵ_1 decreases as the recovery time increases.

3.5. More than one interruption

Several cycles of creep and recovery were investigated. The following programme of creep and recovery was observed: 1 min creep, 1 min recovery; 2 min creep, 2 min recovery; 4 min creep, 4 min recovery, and the specimen failed

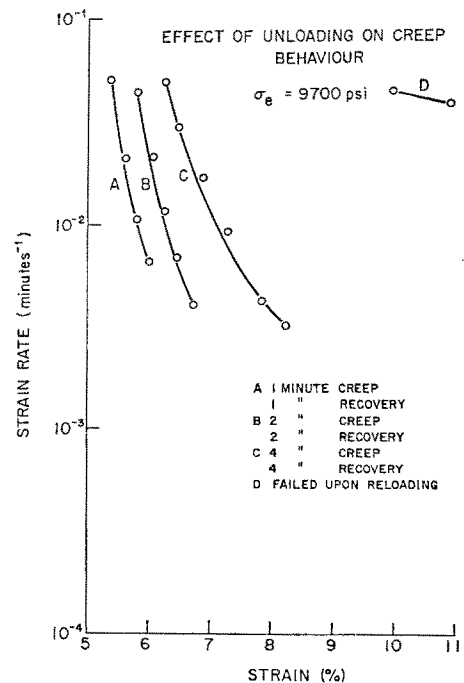


Figure 6 Log strain rate versus strain showing effect of several interruptions.

during the final reloading (Fig. 6). The total strain at the beginning and end of each creep test was as follows: A - 5.3 to 6%, B - 5.8 to 6.8%, C - 6.2 to 8.2%, and in test D failure occurred at about 9 to 10% strain. The initial creep rates were the same for all creep tests. As in the single interruption test, the creep curves all show a decreasing creep rate in accordance with the S-D function, $f(\epsilon)$. Each creep curve corresponds to that part of the function $f(\epsilon)$ which falls within its range of total strain, but each creep curve always starts at the same creep rate. Finally, when the specimen reaches the total

strain corresponding to the minimum in the function, $f(\epsilon)$, the creep rate accelerates and the specimen begins to fail. Thus, each additional interruption further decreased the total time required for failure.

The results of multi-cyclic loading are shown in Fig. 7, where the creep stress was 9200 psi. A cycle consisted of 10 sec creep and 10 sec recovery. Failure, as indicated by the rise in the residual strain after unloading, began after about 10 cycles. If the specimen had been under an uninterrupted stress of 9200 psi it would have failed in about 5×10^4 sec whereas it failed after a

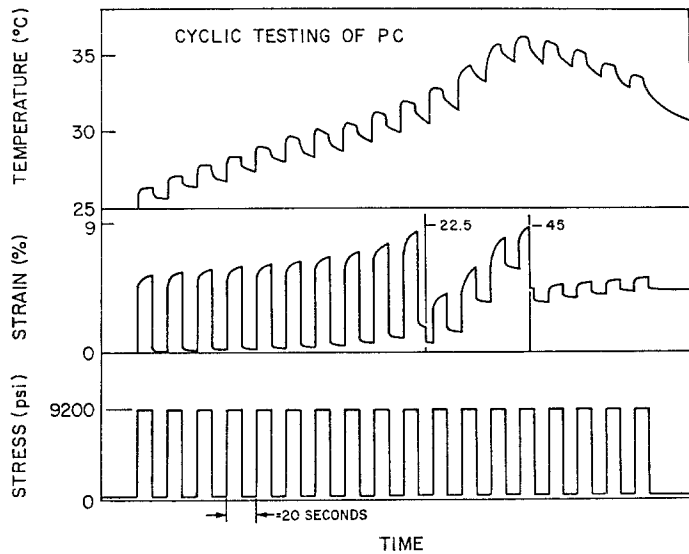


Figure 7 Fatigue test where each cycle consists of 10 sec creep and 10 sec recovery. Temperature, strain, and stress versus time.

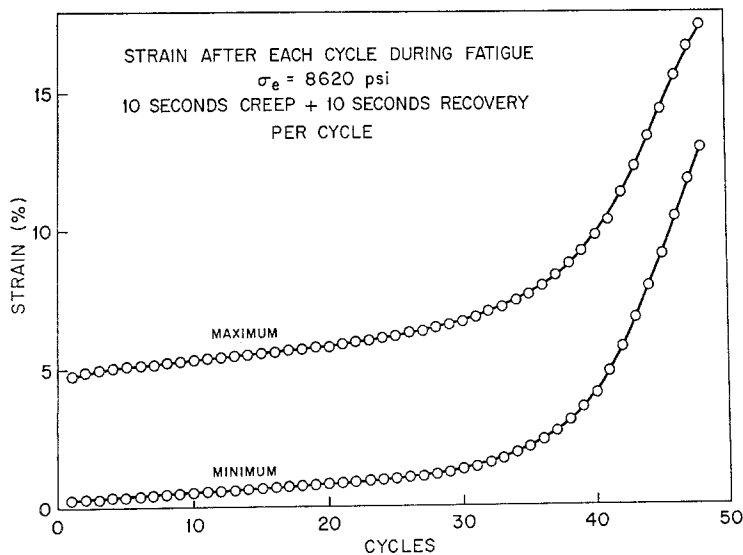


Figure 8 Cumulative strain versus number of cycles. Maximum strain is total strain under stress and minimum strain is total strain during recovery. Curves were observed from type of test shown in Fig. 7.

total time of about 100 sec under stress. Thus, it is seen that each interruption and reloading accelerates the softening process.

Another fatigue test was run where the stress was 8620 psi and each cycle again consisted of 10 sec creep and 10 sec recovery. The results are presented in the form of cumulative strain versus the number of cycles (Fig. 8); one curve represents the maximum strain which occurs prior to unloading and the other curve is the minimum or cumulative persistent strain prior to reloading. The acceleration in the strain occurred at about 25 cycles or after about 4 min of total time under stress. If the creep stress had been uninterrupted, the creep failure would have begun after 10^5 min.

Returning to Fig. 7 it is seen that the temperature is consistently increasing. Under these conditions the temperature rise is about 7°C when failure begins after about the tenth cycle. A 7°C rise in temperature produces a significant increase in the creep rate. Therefore, failure cannot be attributed only to the softening effect. No doubt the frequency of the fatigue test is important in that it determines the rate of temperature rise and also determines how much recovery takes place. A detailed study of the frequency effect was not made.

4. Discussion

The method of failure that has been investigated may be termed excessive deformation or accelerated creep and involves bulk shear. It is useful to be able to predict the time for failure which we call the fatigue life, but it is meant to include uninterrupted creep tests, interrupted creep tests, and periodic cyclic loading. The fatigue life depends on the following variables: stress, creep time, recovery time, the number of interruptions or cycles, the temperature change, and the amount and type of strain in the specimen. This investigation has sampled only a small fraction of the domain of the above variables and has concentrated on the region of high stress. The major observation of our investigation is the fact that interrupting the creep test usually decreases the fatigue life. It is important to try to understand how the process of unloading and loading decreases the fatigue life. It is suggested that the effects of interrupting the creep test, in cyclic loading, can be best understood in terms of the simple creep and recovery test (Figs. 2 and 4).

4.1. The interruption effect

As pointed out by M-B [1] it is helpful to consider two types of deformation: (1) the recoverable strain and (2) the persistent strain. The recoverable part of the strain is associated with the decrease in creep rate that is observed prior to the beginning of failure. M-B [1] viewed the accumulation of recoverable strain as being associated with an internal stress which reduces the creep rate, and they showed that both the recoverable part of the creep strain and the recovery processes could be described as follows:

$$\dot{\epsilon} = \text{const } \chi e^{-Q/kT} e^{\sigma_H v_p/kT} \sinh[(\sigma - \sigma_{\text{int}})v_s/kT] \quad (2)$$

where Q is the activation energy, σ_H is the hydrostatic component of the stress, σ is the applied stress, v_p and v_s are constants, and σ_{int} , the internal stress, is proportional to the recoverable part of the strain. Thus, when σ becomes zero during recovery, the rate of recovery is governed by the internal stress which is proportional to the as yet unrecovered strain. Equation 2 tells why the creep rate is always very fast immediately after reloading. As shown by Figs. 2, 4 and 7 most of the strain is recovered within 1 sec of unloading. Therefore, when a specimen is unloaded and reloaded, the internal stress is reduced nearly to zero so that the initial creep rate for each cycle is always high and the same as for the first cycle.

In addition to the recoverable strain there is a persistent strain which involves a softening process, which increases with the amount of persistent strain. The generation of persistent strain accompanies the generation of the recoverable strain, and the faster the overall creep rate the faster the rate of softening. Also, the greater the total strain, the faster the rate of softening [1]. The net creep rate is controlled by the build up of the internal stress via recoverable strain and by the rate of softening via the persistent strain. At the beginning of a creep cycle the net creep rate decreases as long as the rate of increase in the internal stress exceeds the rate of softening. The beginning of accelerative creep corresponds to the rate of softening balancing the rate of increase in the internal stress. The total strain at the beginning of failure corresponds to the minimum in the S-D plot. This strain was independent of stress for the uninterrupted creep test but varied somewhat with the number of interruptions or cycles. The total strain to failure occurs within a fairly

narrow range which goes from 8.8% for the uninterrupted tests to about 10% for the interrupted test, so that an approximate, but useful, quantitative prediction of failure can be obtained.

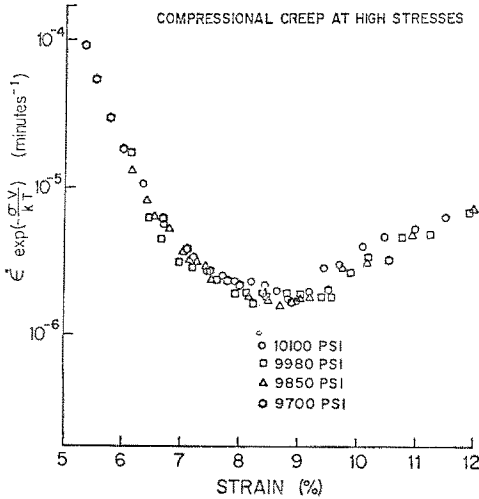


Figure 9 A synthesis of Fig. 1 showing creep rate equals function of stress times a function of the total strain.

4.2. Predicting fatigue life

M-B [1] stated that time to creep failure be given by

$$t_f = \int_{\epsilon_0}^{\epsilon_{min}} \frac{d\epsilon}{\dot{\epsilon}} \quad (3)$$

where ϵ_{min} is the creep strain at the minimum creep rate and ϵ_0 is the instantaneous strain during loading. In the regime of high stresses and for an uninterrupted test

$$t_f^u = \text{constant } e^{Q/kT} e^{-\sigma v/kT} \int_{\epsilon_0}^{\epsilon_{min}} \frac{d\epsilon}{f(\epsilon)} \quad (4)$$

where $f(\epsilon)$ is the function in Equation 1 and can be obtained from the S-D plot shown in Fig. 9. For an interrupted creep test t_f^i , the time to failure, may be designated as the total time during which the load is applied prior to an acceleration in the creep rate

$$t_f^i = \int_{\epsilon_0}^{\epsilon_1} \frac{d\epsilon}{\dot{\epsilon}} + \int_{\epsilon_2}^{\epsilon_{min}} \frac{d\epsilon}{\dot{\epsilon}} \quad (5)$$

where ϵ_0 to ϵ_1 is the range of strain during the first creep test and ϵ_2 to ϵ_{min} the range during the second creep test. The difference between ϵ_2 , the strain immediately after reloading, and ϵ_1 , the strain prior to unloading, is determined by the amount of recovery. The creep rate during the first creep test is in accord with Equation 4.

However, the creep rate during the second creep test is modified in accordance with the observation that the initial creep rate immediately after reloading is the same as the initial creep rate for the first creep test. Thus

$$\dot{\epsilon}(\text{second}) = \frac{\dot{\epsilon}(\epsilon_0)}{\dot{\epsilon}(\epsilon_2)} \cdot \dot{\epsilon}(\text{first}) \quad (6)$$

Thus, after an interruption, the creep rate is increased by the factor $\dot{\epsilon}(\epsilon_0)/\dot{\epsilon}(\epsilon_2)$. The time to failure after an interrupted test is

$$t_f^i = \text{constant } e^{Q/kT} e^{-\sigma v/kT}$$

$$\left[\int_{\epsilon_0}^{\epsilon_1} \frac{d\epsilon}{f(\epsilon)} + \frac{f(\epsilon_2)}{f(\epsilon_0)} \int_{\epsilon_2}^{\epsilon_{min}} \frac{d\epsilon}{f(\epsilon)} \right] \quad (7)$$

The effect of interrupting the creep test on the time to failure is best displayed by comparing the time to failure for the creep test after the interruption with the time required to go from the strain ϵ_1 to ϵ_{min} for the uninterrupted test. The ratio of these times is given by

$$\frac{\Delta t_f^u}{\Delta t_f^i} = \frac{f(\epsilon_0) \int_{\epsilon_1}^{\epsilon_{min}} \frac{d\epsilon}{f(\epsilon)}}{f(\epsilon_2) \int_{\epsilon_2}^{\epsilon_{min}} \frac{d\epsilon}{f(\epsilon)}} \quad (8)$$

The function $f(\epsilon)$ is obtained from Fig. 9; using the Mean Value Theorem,

$$\frac{\Delta t_f^u}{\Delta t_f^i} = \frac{f(\epsilon_0) (\epsilon_{min} - \epsilon_1)}{f(\epsilon_2) (\epsilon_{min} - \epsilon_2)} \cdot \frac{\left[\frac{1}{f(\epsilon)} \right]_{av}^u}{\left[\frac{1}{f(\epsilon)} \right]_{av}^i} \quad (9)$$

Thus the ratio of the additional time to failure without an interruption to the time with an interruption is increased by the rapid creep rate that always occurs immediately after reloading and decreased by the recovery processes which causes $(\epsilon_{min} - \epsilon_2)$ to be greater than $(\epsilon_{min} - \epsilon_1)$. Equation 9 is useful because, given the S-D plot (Fig. 9), knowing the amount of strain prior to interrupting the creep test and the strain after recovery, the additional time to failure can be calculated.

In Fig. 2, ϵ_0 , ϵ_1 and ϵ_2 were 5.8, 7.3 and 7.0% respectively. Using Equation 9 in conjunction with Fig. 9, the theoretical value of $\Delta t_f^u/\Delta t_f^i = 6.8$. Since the specimen failed 4.6 min after being reloaded and since the test would require 33 additional minutes for failure had there been no interruption, the experimental value of $\Delta t_f^u/\Delta t_f^i$ is 7.2. The agreement between the predicted and experimental value is sufficiently close to encourage the use of Equation 9 for

predicting the static fatigue life of a PC after the creep test has been interrupted. It is useful to develop an equation for predicting time to failure after many interruptions which is also called the dynamic fatigue life.

4.3. Predicting cyclic fatigue life

For many cycles the time to failure is given by

$$t_f = \text{constant} \times e^{Q/kT} e^{-\sigma v/kT} \left[\int_{\epsilon_0}^{\epsilon_1} \frac{d\epsilon}{f(\epsilon)} + \left[\frac{f(\epsilon_2)}{f(\epsilon_0)} \int_{\epsilon_2}^{\epsilon_3} \frac{d\epsilon}{f(\epsilon)} + \frac{f(\epsilon_4)}{f(\epsilon_0)} \int_{\epsilon_4}^{\epsilon_5} \frac{d\epsilon}{f(\epsilon)} + \dots + \frac{f(\epsilon_{\min-1})}{f(\epsilon_0)} \int_{\epsilon_{\min-1}}^{\epsilon_{\min}} \frac{d\epsilon}{f(\epsilon)} \right] \right] \quad (10)$$

In Equation 10 it is assumed that the stress and temperature are constant. If the temperature changes, as is likely during a fatigue test, then the temperature factor must be included within the integral for each cycle. ϵ_{\min} appears to be somewhat insensitive to the number of cycles. From our observations ϵ_{\min} ranges from 8.8% to about 10%. A lower limit for t_f for cyclic loading can be calculated relative to t_f^u . The lower limit occurs when the recovery time is so short that all strain softening is cumulative and when the creep cycle is also very short, so that the creep rate does not decrease appreciably by the build up of internal stresses. This is the high frequency condition which also leads to an increase in temperature because the thermal effects are asymmetrical, i.e. the temperature rise during compressive loading exceeds the reduction during unloading. The calculation will, however, assume the temperature is constant. If the creep part of the cycle is very short then the increment of time per creep cycle is as follows:

$$\Delta t_f^n = \frac{f(\epsilon_n)}{f(\epsilon_0)} \int_{\epsilon_n}^{\epsilon_{n+1}} \frac{d\epsilon}{f(\epsilon)} \approx \frac{1}{f(\epsilon_0)} \int_{\epsilon_n}^{\epsilon_{n+1}} d\epsilon = \frac{\Delta \epsilon_n}{f(\epsilon_0)} \quad (11)$$

where $\Delta \epsilon_n$ is the increment of creep strain during the n th cycle.

$$t_f = \sum \Delta t_f^n = \frac{\text{constant} e^{Q/kT} e^{-\sigma v/kT}}{f(\epsilon_0)} \left[\sum \epsilon_n \right] \quad (12)$$

$\Sigma \epsilon_n$ is the cumulative creep strain to failure and

$$\Sigma \epsilon_n = \epsilon_{\min} + N \epsilon_A - \epsilon_0 \quad (13)$$

where ϵ_{\min} is the net strain in the specimen at the beginning of failure and ranges from about 8.8

to 10%, ϵ_A is the difference between the instantaneous strain during loading and reloading, and N is the number of cycles to failure. For PC $\epsilon_A < \Delta \epsilon_n$, and therefore $N \epsilon_A < \epsilon_{\min}$. $N \epsilon_A$ will be neglected in order to establish a lower bound, $t_f(\text{l.b.})$, which is

$$\frac{t_f(\text{l.b.})}{t_f^u} = \frac{(\epsilon_{\min} - \epsilon_0)/f(\epsilon_0)}{\int_{\epsilon_0}^{\epsilon_{\min}} \frac{d\epsilon}{f(\epsilon)}} \quad (14)$$

Using the mean value theorem

$$\frac{t_f^u}{t_f(\text{l.b.})} = f(\epsilon_0) \left[\frac{1}{f(\epsilon)} \right]_{\text{av}} \quad (15)$$

The average value of $[1/f(\epsilon)]$ can be obtained from the S-D plot (Fig. 9) over the range of strain from 5.4 to ϵ_{\min} (8.8%). Thus,

$$\frac{t_f^u}{t_f(\text{l.b.})} = 29 \quad (15a)$$

Equation 15 will depend somewhat on stress in that the instantaneous strain from loading, ϵ_0 , depends on the stress. However, $f(\epsilon_0)$ increases as ϵ_0 decreases and $[1/f(\epsilon)]_{\text{av}}$ decreases as ϵ_0 decreases so that their product should not vary greatly with stress.

The experimental results of Fig. 7 give $t_f^u/t_f = 300$. This discrepancy with the theoretical value of 29 may be mostly related to the 7°C rise in temperature which occurred during the cyclic experiment. The theory is also rudimentary in that it assumes that the rate of softening as a function of strain is the same under a constant stress as under cyclic loading. Also the temperature rise may increase the rate of softening. Feltner [8] showed that for PMMA $t_f^u/t_f > 100$ and also stated that there is an intrinsic effect of softening from cycling the stress.

4.4. Unanswered problems

Let us consider the question of what constitutes the maximum time that the polymer can be exposed to a given stress before it fails. Can the total time under stress be increased beyond t_f^u by interrupting the creep test? Two factors must be considered: (1) the amount of strain that is recovered and (2) the amount of persistent softening. If the increment of total strain during the creep cycle is sufficiently small so that no softening occurs and if the recovery cycle is sufficiently long to recover all the strain, then the state of the specimen would be reversible and the total time for failure would be infinite. At the present time there is the closely related question

as to whether the endurance limit for cyclic loading is greater or less than the static fatigue limit. Bastenev and Zuyev [9] have indicated that in the case of rubber the time-to-failure under cyclic loading is equal or less than that for static fatigue. However, Bastenev and Zuyev did not consider asymmetric cyclic loading where the recovery part of the cycle might be long compared to the creep part. More experimental data at lower stresses are required to answer the above questions. It should also be kept in mind that the above discussion involves compressive loading so that the question of fracture is not involved.

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